

ECON 747 – LECTURE 11:
THE FINANCIAL ACCELERATOR IN A QUANTITATIVE
BUSINESS CYCLE MODEL

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MOTIVATION

- ▶ Previous lecture: [Bernanke and Gertler \(1989\)](#) [“BG”] as a classic CSV model
 - ▶ Countercyclical strength of agency frictions gives rise to endogenous amplification
 - ▶ Transmission through borrower net worth
- ▶ This lecture: [Bernanke, Gertler, and Gilchrist \(1999\)](#) [“BGG”]
 - ▶ Take a *quantitative* standpoint
 - ▶ Add nominal rigidities and decision lags in investment
- ▶ Also highlight a number of other contributions that stand in the CSV tradition

BGG 1999

MOTIVATION OF BGG

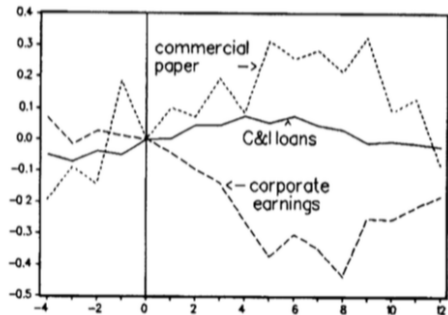
- ▶ BGG essentially motivate their work along two lines:
 1. Credit market frictions could help explaining “garden variety” cyclical fluctuations
 - ▶ Endogenous amplification is possible, as shown e.g. in BG
 2. Mounting empirical evidence that credit frictions are relevant
 - ▶ Investment literature, e.g. [Fazzari, Hubbard, and Petersen \(1988\)](#)
 - ▶ Empirical macro papers, e.g. earlier work by the same authors: [Bernanke, Gertler, and Gilchrist \(1996\)](#)
- ▶ From our standpoint, we should bear in mind that this is work from well before the financial crisis of 2008/09 ...

BGG (1996, RESTAT): EMPIRICAL EVIDENCE

- ▶ Look at episodes of monetary tightening (as defined by Romer and Romer)
- ▶ Consider as proxies for credit constraints the difference between:
 1. Bank loans vs. commercial paper
 2. Credit to large firms vs. small firms

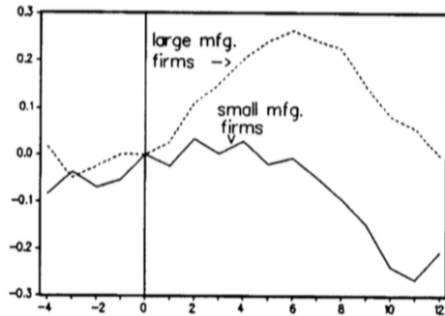
BGG (1996, RESTAT): EMPIRICAL EVIDENCE

Commercial Paper, C&I Loans, and Corporate Earnings



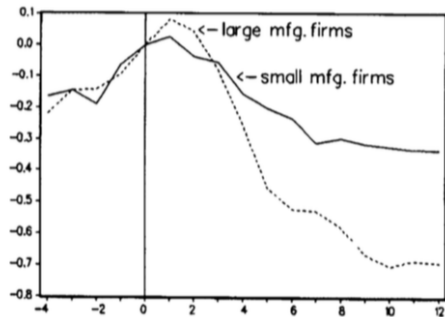
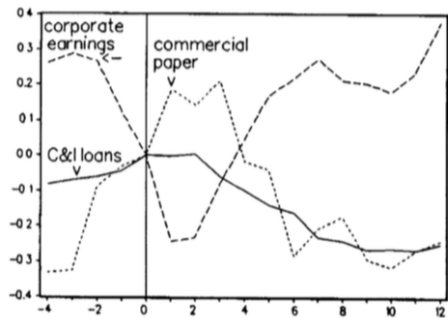
Short-Term Debt of Manufacturing Firms

1968:4

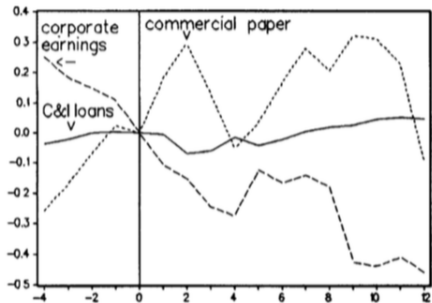


BGG (1996, RESTAT): EMPIRICAL EVIDENCE

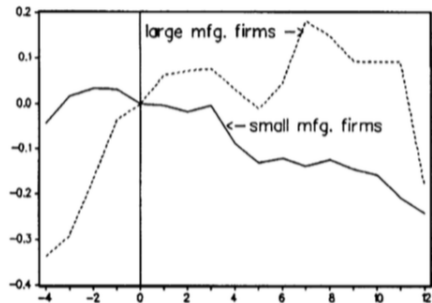
1974:2



BGG (1996, RESTAT): EMPIRICAL EVIDENCE

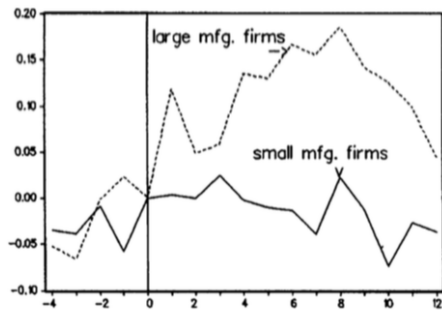
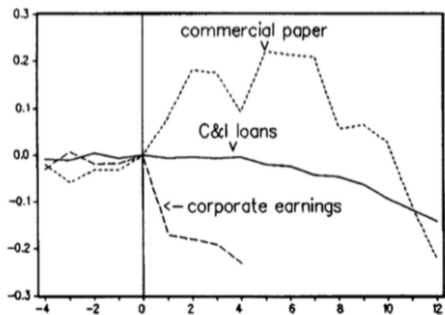


1979:4



BGG (1996, RESTAT): EMPIRICAL EVIDENCE

1988:4



IDEA OF BGG 1999 MODEL

- ▶ Embed CSV friction into a New Keynesian framework
- ▶ Why New Keynesian?
 - ▶ Has become a widely accepted framework for quantitative business cycle questions
 - ▶ Can study monetary policy
- ▶ Remember the key New Keynesian ingredients
 - ▶ Monopolistic competition
 - ▶ Sticky prices

IDEA OF BGG 1999 MODEL

- ▶ BGG explain two challenges of adding CSV into New Keynesian environment:
 1. Need lending and borrowing to occur, so need to introduce some heterogeneity
 2. Derive financial contracts from first principals, lay out the market incompleteness
- ▶ These are the very ingredients necessary for financial frictions to matter, as highlighted throughout this course

THE MODEL: OUTLINE

- ▶ Proceed in three steps
 1. Overview: agents and objectives, highlight some comparison with BG
 2. Describe agency friction: contracting problem in partial equilibrium
 3. Embed friction into New Keynesian framework, general equilibrium analysis

THE MODEL: OVERVIEW

- ▶ Discrete time, runs infinitely
- ▶ Three agents: households, entrepreneurs and retailers
- ▶ Households and entrepreneurs have similar roles as in BG
- ▶ Retailers are present to introduce nominal rigidities without complicating the entrepreneur's problem
 - ▶ Entrepreneurs become wholesalers who sell inputs to retailers
- ▶ There is a government which runs fiscal and monetary policy

THE MODEL: OVERVIEW

- ▶ Households work, consume and save
 - ▶ Relative to BG, they live infinitely and make a labor supply decision
- ▶ Entrepreneurs are risk-neutral
- ▶ Entrepreneurs die with probability γ , which gives expected live span $\frac{1}{1-\gamma}$
 - ▶ “capture phenomenon of ongoing births and deaths of firms”
 - ▶ “preclude possibility that the entrepreneurial sector will ultimately accumulate enough wealth to be fully self financing”
- ▶ Entrepreneurs acquire physical capital in t , produce with capital and labor in $t + 1$, supply labor inelastically and consume

THE MODEL: OVERVIEW

- ▶ Acquisition of capital by entrepreneurs is financed from net worth and borrowing
- ▶ Net worth comes from two sources:
 1. Supplying labor inelastically (as in BG)
 2. Profits, incl. capital gains accumulated from previous capital investment (not in BG)
- ▶ Higher net worth mitigates agency problems associated with external finance and reduces the external finance premium faced by entrepreneurs in equilibrium
 - ▶ The agency problem will be discussed below

THE MODEL: OVERVIEW

- ▶ Retailers buy the goods that entrepreneurs produce, differentiate them and sell them to households
- ▶ They face monopolistic competition, so that they become price setters (face downward sloping demand curves)
- ▶ Their price setting is subject to stickiness

THE MODEL: AGENCY FRICTION

- ▶ Begin with entrepreneurs' investment & contracting problem in partial equilibrium
 - ▶ Price of capital and the marginal return on capital are taken as given for now
 - ▶ Similar way of proceeding as in BG
- ▶ Assume only one-period contracts are possible
 - ▶ E.g. because markets are sufficiently anonymous
 - ▶ Is this a good assumption?

THE MODEL: MORE DETAILS ON ENTREPRENEUR

- ▶ At time t , entrepreneur j purchases a homogeneous capital good at price Q_t
- ▶ Return between t and $t + 1$ is $\omega^j R_{t+1}^k$, features idiosyncratic and aggregate risk
 - ▶ Idiosyncratic risk ω^j is iid across time and entrepreneurs
 - ▶ Distributed with cdf $F(\omega)$ and mean $\mathbb{E}(\omega^j) = 1$
- ▶ Important: setting differs from BG
 - ▶ Cost of undertaking is the same across entrepreneurs, because capital is homogeneous, all entrepreneurs face price Q_t
 - ▶ But return is different across j

THE MODEL: MORE DETAILS ON ENTREPRENEUR

- ▶ Entrepreneurs finance purchases of capital goods with net worth N and by borrowing the amount B :

$$Q_t K_{t+1}^j = N_{t+1}^j + B_{t+1}^j$$

- ▶ Here $t + 1$ denotes end-of-period t quantities
- ▶ B_{t+1} is provided by households, who face an opportunity of funds given by the riskless rate of return R_{t+1}

THE MODEL: AGENCY FRICTION

- ▶ CSV problem: only entrepreneur j can observe ω^j
- ▶ Lenders must pay an auditing cost
- ▶ In BG this is a constant expressed in units of capital (γ in the BG notation)
- ▶ Here it is a share μ of the realized gross return:

$$\mu \omega^j R_{t+1}^k Q_t K_{t+1}^j$$

THE MODEL: AGENCY FRICTION

- ▶ We start by abstracting from aggregate risk
 - ▶ Not only are the return and price of capital given, for we assume they are certain
- ▶ The setting described above gives rise to an optimal contract
- ▶ This optimal contract is a **risky debt contract**
 - ▶ Features a constant repayment, independent of the project outcome (in case of no default)
 - ▶ It is risky contract as it features default

THE MODEL: AGENCY FRICTION

- ▶ Take as given $Q_t K_{t+1}^j$, B_{t+1}^j and R_{t+1}^k
- ▶ The contract is characterized by:
 - ▶ A gross non-default loan rate Z_{t+1}^j
 - ▶ A threshold value for idiosyncratic risk $\bar{\omega}^j$ above which the loan is repaid
- ▶ The threshold value is given by

$$\bar{\omega}^j R_{t+1}^k Q_t K_{t+1}^j = Z_{t+1}^j B_{t+1}^j$$

THE MODEL: AGENCY FRICTION

- ▶ For $\omega^j \geq \bar{\omega}^j$:
 - ▶ Lender gets $Z_{t+1}^j B_{t+1}^j$
 - ▶ Entrepreneur gets $\omega^j R_{t+1}^k Q_t K_{t+1}^j - Z_{t+1}^j B_{t+1}^j$
- ▶ For $\omega^j < \bar{\omega}^j$, default:
 - ▶ Lender gets $(1 - \mu)\omega^j R_{t+1}^k Q_t K_{t+1}^j$
 - ▶ Entrepreneur gets 0

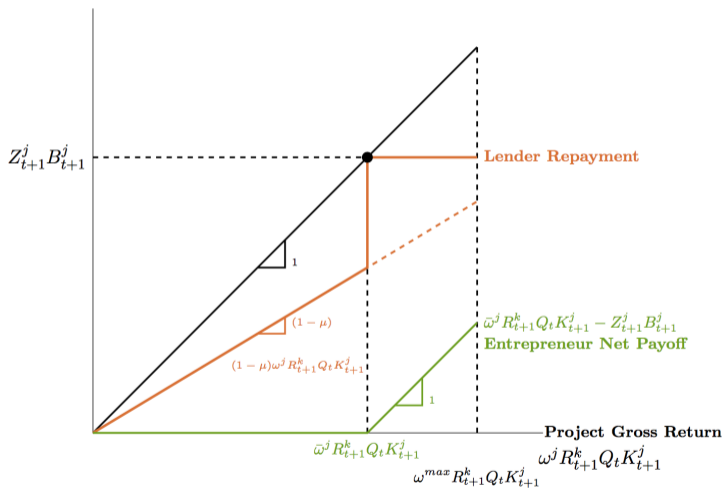
THE MODEL: AGENCY FRICTION

- ▶ The loan contract must satisfy lender participation (***)

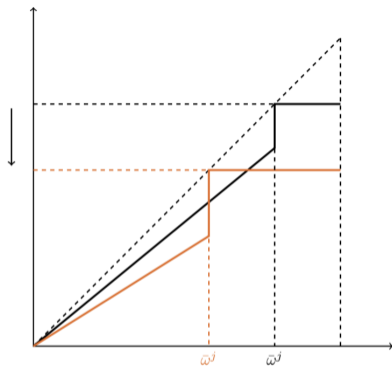
$$\begin{aligned} [1 - F(\bar{\omega}^j)] Z_{t+1}^j B_{t+1}^j + (1 - \mu) \int_0^{\bar{\omega}^j} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) \\ = R_{t+1} B_{t+1}^j \end{aligned}$$

- ▶ Tradeoff: the higher $\bar{\omega}^j$...
 - ▶ The higher the non-default payoff
 - ▶ The larger the default region
- ▶ Under assumptions on $F(\omega)$ and right model parameterization there is a unique interior $\bar{\omega}^j$ which maximizes expected return

RISKY DEBT CONTRACT



PREVIEW OF MECHANICS



- ▶ If net worth \uparrow
- ▶ Less borrowing necessary: $B_{t+1}^j \downarrow = Q_t K_{t+1}^j - N_{t+1}^j \uparrow$
- ▶ Lowers default probability, decreases lender's non-default payoff

THE MODEL: AGENCY FRICTION

- ▶ So far: contract in partial equilibrium, abstracting from aggregate risk
- ▶ We now stay in partial equilibrium but add aggregate risk
 - ▶ This means the price and return of capital are taken as given, but are uncertain
- ▶ Still take as given $Q_t K_{t+1}^j$ and B_{t+1}^j

THE MODEL: AGENCY FRICTION

- ▶ With aggregate risk the contract generalizes easily, due to risk neutrality of the entrepreneur
- ▶ $\bar{\omega}^j$ will now generally depend on the ex-post realization R_{t+1}^k
- ▶ Since entrepreneur is risk neutral, she only cares about the mean return and is willing to bear all aggregate risk
- ▶ Agents contract contingent on aggregate state, gives a state-contingent non-default repayment
- ▶ In other words, there is a set of of contracts, one for each realization of R_{t+1}^k

NET WORTH AND OPTIMAL CHOICE OF CAPITAL

- ▶ Given the contracting setting we now turn to the entrepreneur's general problem of choosing K_{t+1}^j (and B_{t+1}^j)
- ▶ The entrepreneur's payoff is

$$\mathbb{E} \left\{ \int_{\bar{\omega}^j}^{\infty} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) - (1 - F(\bar{\omega}^j)) Z_{t+1} B_{t+1} \right\}$$

- ▶ Using the relation $\bar{\omega}^j R_{t+1}^k Q_t K_{t+1}^j = Z_{t+1}^j B_{t+1}^j$, we get

$$\mathbb{E} \left\{ \int_{\bar{\omega}^j}^{\infty} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) - (1 - F(\bar{\omega}^j)) \bar{\omega}^j R_{t+1}^k Q_t K_{t+1}^j \right\}$$

NET WORTH AND OPTIMAL CHOICE OF CAPITAL

- ▶ Problem reduces to choosing K_{t+1}^j and $\bar{\omega}^j$ to maximize the objective on the previous slide, subject to the constraint that the optimal contract satisfies (***)
- ▶ The optimality condition for this problem can be shown to be of the following form

$$Q_t K_{t+1}^j = \psi(s_t) N_{t+1}^j$$

with $s_t \equiv \mathbb{E}(R_{t+1}^k / R_{t+1})$, $\psi(1) = 1$, $\psi' > 0$

NET WORTH AND OPTIMAL CHOICE OF CAPITAL

$$Q_t K_{t+1}^j = \psi(s_t) N_{t+1}^j$$

- ▶ Key relationship in the model
- ▶ Capital expenditures are proportional to the net worth of the entrepreneur
- ▶ Proportionality factor is increasing in the expected discounted return to capital s_t
 - ▶ All else equal, if $s_t \uparrow$, the default probability is reduced, so entrepreneur can take on more debt and buy more capital
 - ▶ The increase in capital will be constrained by the fact that expected default costs also rise as the ratio of borrowing to net worth increases
- ▶ The linear relation facilitates aggregation across entrepreneurs

NET WORTH AND OPTIMAL CHOICE OF CAPITAL

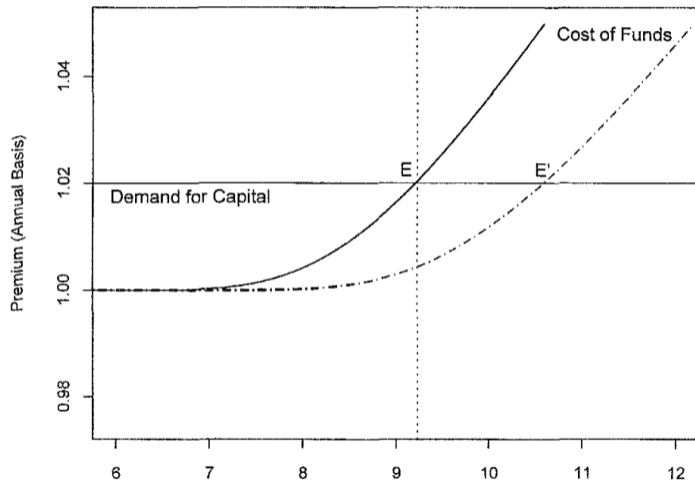
- ▶ An equivalent way of expressing the condition is

$$\mathbb{E}(R_{t+1}^k) = s \left(\frac{N_{t+1}^j}{Q_t K_{t+1}^j} \right) R_{t+1}$$

with $s' < 0$

- ▶ Equilibrium marginal return on capital must equal the marginal cost of external finance
- ▶ External finance is given by the safe rate, scaled up by a premium term $s \left(N_{t+1}^j / Q_t K_{t+1}^j \right)$
- ▶ Premium depends inversely on the share financed by entrepreneur's own capital

NET WORTH AND OPTIMAL CHOICE OF CAPITAL



NET WORTH AND OPTIMAL CHOICE OF CAPITAL

- ▶ The chart shows the cost of funds curve $s(\cdot)$: the return relative to the risk-free rate as a function of K
- ▶ This is plotted for a given Q and given N
- ▶ If N is increased, the curve shifts out: the cost required to finance a given amount of capital falls
- ▶ We are still characterizing a partial equilibrium in which the return on capital is given (in this case by 1.02 relative to the risk-free rate)
- ▶ Therefore for a given return more capital can be financed when net worth rises

GENERAL EQUILIBRIUM: NEW KEYNESIAN MODEL

- ▶ We now embed the partial equilibrium contracting problem into a New Keynesian (NK) structure
- ▶ R_{t+1} , R_{t+1}^k and Q_t will be endogenous
- ▶ Focus on describing the entrepreneur problem, since the household and retailer problems will be standard

FULL NK MODEL: ENTREPRENEURS

- ▶ The return on capital results from the fact that capital can be used in production

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

- ▶ Here $L_t = H_t^\Omega (H_t^e)^{1-\Omega}$, where
 - ▶ H_t is labor endogenously supplied by households
 - ▶ H_t^e is labor inelastically supplied by entrepreneurs (will be normalized to 1)
- ▶ Like in BG, this links the available resources to the state of the economy via wages
 - ▶ As capital does not fully depreciate, resources also affected by previous return

FULL NK MODEL: ENTREPRENEURS

- ▶ Capital is accumulated as follows

$$K_{t+1} = \Phi \left(\frac{I_t}{K_t} \right) K_t + (1 - \delta)K_t$$

- ▶ The presence of adjustment costs gives the familiar variation in the price of capital in equilibrium (Tobin's Q)

$$Q_t = \left[\Phi' \left(\frac{I_t}{K_t} \right) \right]^{-1}$$

FULL NK MODEL: ENTREPRENEURS

- ▶ Entrepreneurs sell their output to retailers, at the relative price $1/X_t$, which they take as given
- ▶ This means that the rent paid to one unit capital is given by

$$\frac{1}{X_t} \alpha \frac{Y_{t+1}}{K_{t+1}}$$

- ▶ The capital Euler equation gives is the expected gross return on capital

$$\mathbb{E}_t(R_{t+1}^k) = \mathbb{E}_t \left[\frac{\frac{1}{X_t} \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)Q_{t+1}}{Q_t} \right]$$

FULL NK MODEL: ENTREPRENEURS

- ▶ We can derive the aggregate supply of funds from the solution to the contracting problem for entrepreneur j
- ▶ Linearity in $Q_t K_{t+1}^j = \psi(s_t) N_{t+1}^j$ allows to sum over j to get

$$Q_t K_{t+1} = \psi(s_t) N_{t+1}$$

which we can invert to the relation

$$\mathbb{E}(R_{t+1}^k) = s \left(\frac{N_{t+1}}{Q_t K_{t+1}} \right) R_{t+1}$$

that we have studied above

FULL NK MODEL: ENTREPRENEURS

- ▶ Aggregate entrepreneurial net worth is given by wage income as well as by equity holdings V_t (from past returns on capital) of entrepreneurs that do not die

$$N_{t+1} = \gamma V_t + W_t^e$$

- ▶ Entrepreneurs who die just consume their equity share

$$C_T^e = (1 - \gamma)V_t$$

FULL NK MODEL: ENTREPRENEURS

- ▶ Equity holdings V_t are given by the gross return on capital, net of repayments to the lender
- ▶ This is the aggregation of the objective function of an individual entrepreneur in the contracting problem above (combined with lender break even condition)
- ▶ V_t is equal to

$$R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}} \right) (Q_{t-1} K_t - N_t)$$

- ▶ You will derive this expression in Assignment 4

FULL NK MODEL: ENTREPRENEURS

- ▶ In the above expression, the term

$$\frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}}$$

captures the premium for external finance

- ▶ Spread between the total rate per unit of external funds and the risk free rate R_t

FULL NK MODEL: ENTREPRENEURS

- ▶ From $N_{t+1} = \gamma V_t + W_t^e$, the expression of V_t and the equilibrium expression for W_t , we can derive a law of motion for net worth:

$$N_{t+1} = \gamma \left\{ R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}} \right) (Q_{t-1} K_t - N_t) \right\} + (1 - \alpha)(1 - \Omega) A_t K_t^\alpha H_t^{(1-\alpha)\Omega}$$

FULL NK MODEL: ENTREPRENEURS

- ▶ Two key equilibrium equations

1. The supply of funds, derive further above

$$\mathbb{E}(R_{t+1}^k) = s \left(\frac{N_{t+1}}{Q_t K_{t+1}} \right) R_{t+1}$$

2. The evolution of net worth, shown on the previous slide

- ▶ One equation tells us how variation in net worth allows more capital to be financed, the other one how net worth endogenously varies over the business cycle

FULL NK MODEL: HOUSEHOLD AND RETAILERS

- ▶ Households work, consume and save
- ▶ Retailers buy the goods that entrepreneurs produce, differentiate them and sell them to households
 - ▶ Face monopolistic competition and price stickiness (Calvo)
- ▶ The details are omitted here

FULL NK MODEL: DYNAMICS

- ▶ The model is log-linearized
- ▶ Parameters are calibrated
- ▶ Consider the responses to the following shocks
 1. Monetary policy shock
 2. Technology shock
 3. Government expenditure shock
 4. Wealth shock
- ▶ In the case of the monetary policy shock BGG shows some VAR responses as a benchmark

MONETARY POLICY SHOCK IN SVAR

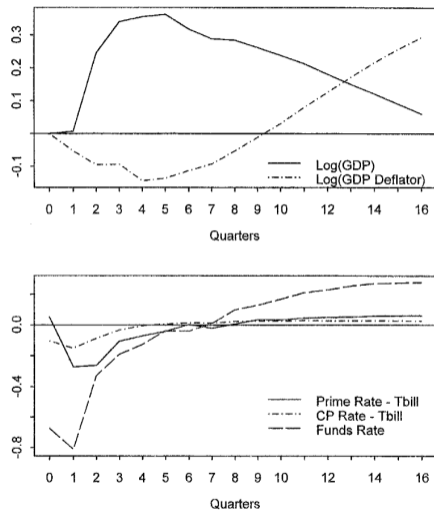


Fig. 2. Impulse response to a funds rate shock.

MODEL DYNAMICS

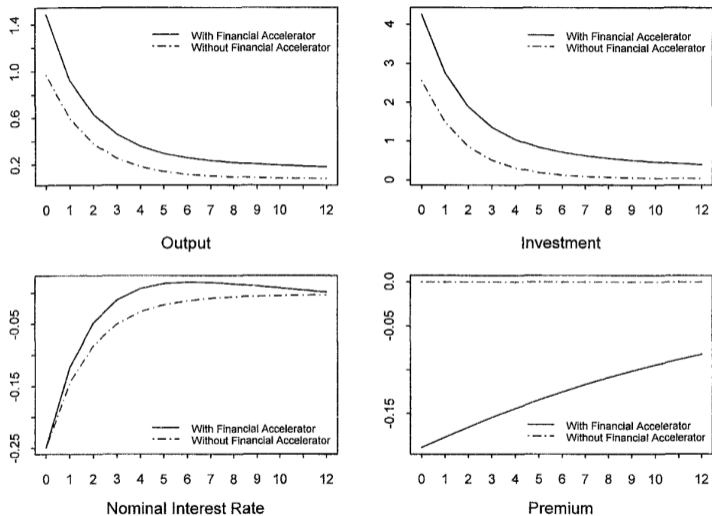
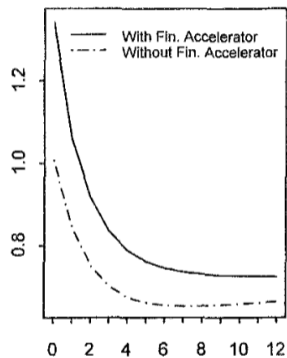
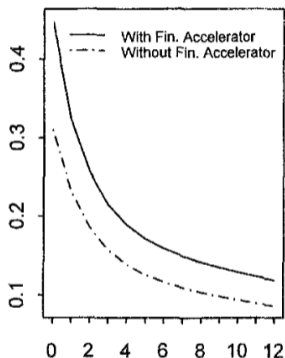


Fig. 3. Monetary shock – no investment delay. All panels: time horizon in quarters.

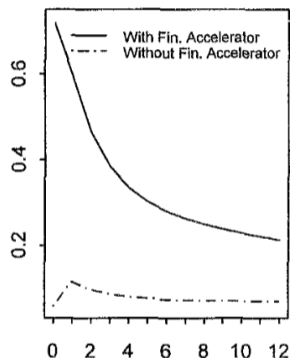
MODEL DYNAMICS



Technology Shock



Demand Shock



Wealth Shock

THE MODEL: EXTENSION WITH INVESTMENT DELAYS

- ▶ Suppose investment is chosen j periods in advance
- ▶ This generates hump shaped responses for output and investment

MODEL DYNAMICS

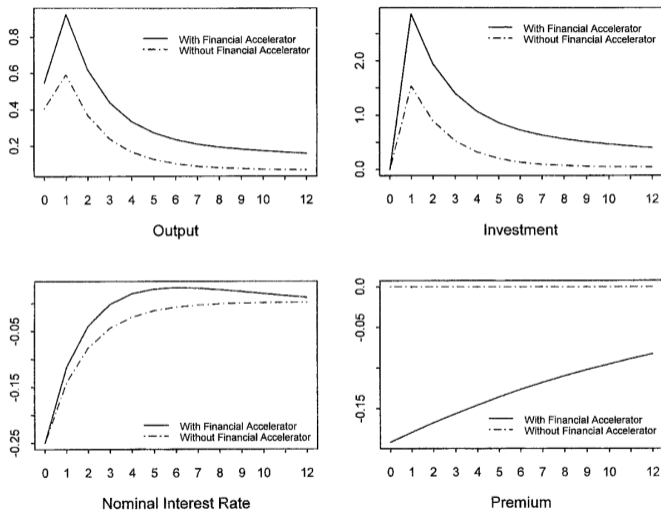


Fig. 5. Monetary shock – one period investment delay. All panels: time horizon in quarters.

THE MODEL: EXTENSION WITH SEVERAL SECTORS

- ▶ Introduce two types of firms, one with easy and one with difficult access to credit (different average premia)

MODEL DYNAMICS

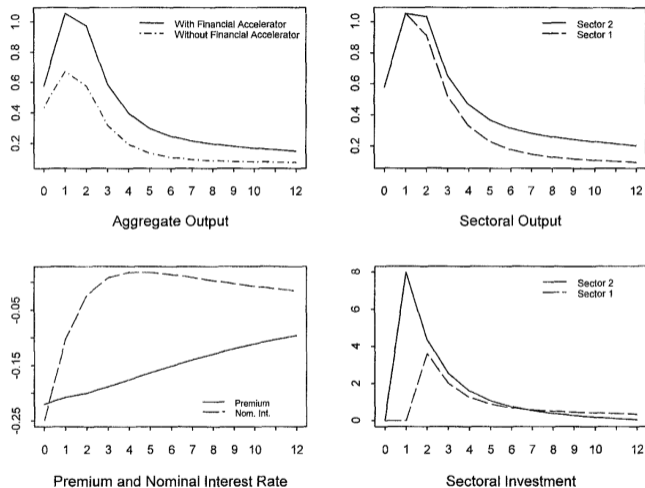


Fig. 6. Monetary shock – multisector model with investment delays. All panels: time horizon in quarters. Aggregate output: models with and without financial accelerator; other panels: model with financial accelerator.

SUMMARY OF INSIGHTS

- ▶ Financial accelerator has significant quantitative influence on business cycles
- ▶ Two-way feedback between credit and real activity at the heart of the model
- ▶ Countercyclical premia in line with what we see in the data
- ▶ Financial friction is relevant in the transmission of various shocks

SUMMARY & TAKING STOCK

A BGG SOLUTION RECIPE: 4 STEPS

1. Characterize optimal contract in partial equilibrium, no risk:

- ▶ Take as given $K_{t+1}^j, B_{t+1}^j, N_{t+1}^j, Q_t, R_{t+1}^k, R_{t+1}$
- ▶ Find Z_{t+1}^j and $\bar{\omega}^j$ (risky debt contract)
- ▶ Solution is given by:
 - (1) Threshold condition for $\bar{\omega}^j$
 - (2) Lender participation (***)

2. Add risk to partial equilibrium contract:

- ▶ Allow R_{t+1}^k to vary, but not endogenously
- ▶ Can write contract contingent on realization of R_{t+1}^k

A BGG SOLUTION RECIPE: 4 STEPS

3. Determine choice of how much to invest:

- ▶ Find K_{t+1}^j, B_{t+1}^j (together with Z_{t+1}^j and $\bar{\omega}^j$)
- ▶ Take as given N_{t+1}^j and Q_t, R_{t+1}^k, R_{t+1}
- ▶ Can substitute out $B_{t+1}^j Z_{t+1}^j$ from problem
- ▶ Get optimality condition: K_{t+1}^j choice is linear in N_{t+1}^j , scale is increasing in $s_t \equiv \mathbb{E}(R_{t+1}^k/R_{t+1})$

4. Characterize general equilibrium:

- ▶ Aggregate up condition from step 3 (linearity!) to get aggregate cost of funds curve as first key equation
- ▶ Q_t, R_{t+1}^k, R_{t+1} determined from production, capital and safe asset choice
- ▶ Obtain aggregate law of motion for N_{t+1} as second key equation

BG1989 vs. BGG1999: SETUP

- ▶ Agents:
 - ▶ BG: HH, entrepreneurs
 - ▶ BGG: HH, entrepreneurs, retailers
- ▶ Borrowing motive:
 - ▶ BG: OLG structure
 - ▶ BGG: entrepreneurs die with constant probability
- ▶ Labor supply:
 - ▶ BG: inelastic for both agents
 - ▶ BGG: choice for HH, inelastic for entrepreneur

BG1989 vs. BGG1999: FRICTION AND CONTRACT

- ▶ Heterogeneity across entrepreneurs:
 - ▶ BG: cost $x(\omega)$
 - ▶ BGG: return $\omega^j R_{t+1}^k$
- ▶ Auditing cost:
 - ▶ BG: γ units of capital
 - ▶ BGG: share μ of project outcome
- ▶ Opportunity cost of funds:
 - ▶ BG: constant r
 - ▶ BGG: endogenous R_{t+1}
- ▶ Optimal contract features:
 - ▶ BG: not a debt contract
 - ▶ BGG: debt contract

BG1989 vs. BGG1999: ANALYSIS

- ▶ General focus:
 - ▶ BG: amplification in stylized model
 - ▶ BGG: role of friction in quantitative model
- ▶ Shocks:
 - ▶ BG: productivity
 - ▶ BGG: several shocks, including monetary policy

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