

ECON 747 – LECTURE 5:
INCOMPLETE MARKETS, HETEROGENEOUS AGENTS
AND PRECAUTIONARY SAVINGS

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Spring 2024

RECAP FROM PREVIOUS LECTURE

- ▶ We built a benchmark complete markets model
- ▶ Heterogeneity (idiosyncratic risk) could be insured away

THIS LECTURE

- ▶ Introduce **heterogeneity** and **incomplete markets** in combination
 - ▶ Recall that we need both for financial frictions to matter
- ▶ In today's lecture, the heterogeneity will consist of **idiosyncratic income risk**, as in the complete markets case; the market incompleteness will be such that agents only have **non-state contingent bonds** and face an **exogenous borrowing limit**
 - ▶ In future lectures, we will have simpler heterogeneity (e.g. two agents) but deal with borrowing limits that are endogenous
- ▶ We will learn today how **precautionary savings** are a consequence of market incompleteness

REFERENCES

- ▶ A good textbook reference for this lecture is the Ljungqvist and Sargent text book *Recursive Macroeconomic Theory* (2nd edition), Chapters 16 and 17
- ▶ As usual, I will provide additional references throughout

OVERVIEW

1. Precautionary savings in partial equilibrium
2. Incomplete markets models: baseline setting
3. Models with incomplete markets, heterogeneous agents and precautionary savings
 - 3.1 Pure credit economy: [Huggett \(1993\)](#)
 - 3.2 Adding capital: [Aiyagari \(1994\)](#)
 - 3.3 Adding aggregate risk: [Krusell and Smith \(1998\)](#)
4. The latest generation of heterogeneous agent models (“HANK”)

PRECAUTIONARY SAVINGS IN PARTIAL EQUILIBRIUM

WHY START IN PARTIAL EQUILIBRIUM?

- ▶ We want to build intuition for the basic decision margins of individual agents
- ▶ Recall Lucas tree model: take consumption patterns as given, price assets (GE)
- ▶ Here: take asset prices as given, study agents' consumption behavior (PE)

A SIMPLE PE SETTING

- ▶ Assume single household, endowment income, no uncertainty
- ▶ Budget constraint and no-Ponzi condition

$$c_t + b_{t+1} = Rb_t + y_t$$
$$\lim_{t \rightarrow \infty} \frac{b_{t+1}}{R_t} = 0$$

- ▶ Euler equation for risk-free bond is

$$u'(c_t) = \beta R u'(c_{t+1})$$

- ▶ Consumption path will be:
 - ▶ Constant if $\beta R = 1$
 - ▶ Increasing if $\beta R > 1$
 - ▶ Decreasing if $\beta R < 1$

THE PERMANENT INCOME HYPOTHESIS

- ▶ In this world, current income has no effect on consumption path, only lifetime income (permanent income) does
- ▶ How to see this formally? Consolidate the budget constraint and impose no-Ponzi condition:

$$Rb_t = c_t - y_t + b_{t+1}$$

$$Rb_t = c_t - y_t + \frac{c_{t+1} - y_{t+1}}{R} + \frac{b_{t+2}}{R}$$

...

$$Rb_t = \sum_{j=0}^{\infty} \frac{c_{t+j} - y_{t+j}}{R^j}$$

THE PERMANENT INCOME HYPOTHESIS

- ▶ Suppose we have log utility and $\beta R = 1$ so that $c_{t+1} = c_t$
- ▶ In that case we get

$$Rb_t = c_t \sum_{j=0}^{\infty} \frac{1}{R^j} + \sum_{j=0}^{\infty} \frac{y_{t+j}}{R^j}$$
$$c_t = \frac{R}{R-1} \left[Rb_t + \sum_{j=0}^{\infty} \frac{y_{t+j}}{R^j} \right]$$

- ▶ Current consumption depends only on lifetime income (asset wealth and all future endowment income)
- ▶ Current income changes (that leave permanent income the same) do not change consumption

TESTING THE PIH EMPIRICALLY

▶ Macro data

- ▶ Hall (1978): assume quadratic utility $\Rightarrow \mathbb{E}_t c_{t+1} = c_t$; no current information (other than current consumption) should predict future consumption; confirmed for disposable income, rejected for stock prices
- ▶ Wilcox (1989): look at pre-announced increases in social security benefit; do not rise consumption immediately, so PIH rejected; possible explanation: liquidity constraints

▶ Micro data

- ▶ Zeldes (1989a): tests for liquidity constraint explanation; PIH holds for “rich” households
- ▶ Shea (1995): consider households with long-term union contracts; consumption responds to predictable wage movements; responds asymmetrically to increase/decreases

PRECAUTIONARY SAVINGS

- ▶ With uncertainty, current consumption depends only on *expected* lifetime income
- ▶ “Precautionary savings” = difference between consumption
 1. when income is certain
 2. when income is uncertain but has same mean as in 1.

PRECAUTIONARY SAVINGS

- ▶ Look at case $\beta R = 1$ and compare certainty to uncertainty case:

$$\begin{aligned}u'(c_t) &= u'(c_{t+1}) \\ u'(c_t^*) &= \mathbb{E}_t [u'(c_{t+1}^*)]\end{aligned}$$

- ▶ Suppose $\mathbb{E}_t [c_{t+1}^*] = c_{t+1}$
- ▶ Precautionary savings would mean $c_t^* < c_t$. Is this the case?

PRECAUTIONARY SAVINGS

- ▶ Use result $\mathbb{E}_t [u'(c_{t+1}^*)] > u'(\mathbb{E}_t [c_{t+1}^*])$
 - ▶ This holds iff $u''' > 0$ (based on Jensen's inequality)
 - ▶ $u''' > 0$ means *marginal utility is convex*
 - ▶ See [Zeldes \(1989b\)](#)

- ▶ Substitute assumption $\mathbb{E}_t [c_{t+1}^*] = c_{t+1}$ into the right hand side of the above inequality

$$\begin{aligned}\mathbb{E}_t [u'(c_{t+1}^*)] &> u'(c_{t+1}) \\ u'(c_t^*) &> u'(c_t) \\ c_t^* &< c_t\end{aligned}$$

(if $u'' < 0$ i.e. function u' is decreasing)

INCOMPLETE MARKETS MODELS: A BASELINE SETTING

ENVIRONMENT

- ▶ Similar to previous lecture, with some changes in notation
- ▶ Households $i = 1, \dots, I$, ex ante identical, but ex-post heterogeneous
- ▶ Preferences:

$$U_{i,t} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{i,\tau})$$

- ▶ **Incomplete markets:** only risk free bonds

$$c_{i,t} + a_{i,t+1} \leq (1+r)a_{i,t} + s_{i,t}w$$

ENVIRONMENT

- ▶ $s_{i,t}$ is employment status or productivity
- ▶ $s_{i,t}$ follows m state Markov process with transition matrix $\mathcal{P}_{m \times m}$ which gives transition probabilities between states k and ℓ as $\mathcal{P}(s_{i,t} = s^k, s_{i,t+1} = s^\ell)$
- ▶ Suppose assets can be chosen from grid $\mathcal{A} = \{a^1, \dots, a^n\}$
(Careful: here the superscript denotes the realization of the state, not the history)
- ▶ No aggregate uncertainty
- ▶ Assume for now that $\beta(1+r) < 1$, treat r and w as parameters

REMARKS

- ▶ Defining s and a over discrete spaces will be helpful when we take about solution methods for these models further below
- ▶ Important note: in this setting, there are actual individual savings (no “illusion of choice”) because there are incomplete markets and heterogeneity

BELLMAN EQUATION OF AGENT i

$$V(a, s^k) = \max_{c, a' \in \mathcal{A}} \left\{ u(c) + \beta \sum_{\ell=1}^m \mathcal{P}(s^k, s^\ell) V(a', s^\ell) \right\}$$

subject to

$$c + a' = (1 + r)a + s^k w$$

(I have dropped subscript i for ease of notation)

WHAT'S THE SOLUTION

- ▶ What are the policy functions in this economy?

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- ▶ With a discrete state space, the policy functions are also transition matrices
- ▶ They are the same for each agent, because agents are ex-ante identical

WEALTH-EMPLOYMENT DISTRIBUTION

- ▶ In this economy, we can define the unconditional distribution of states for a given agent recursively
- ▶ Define $\lambda_t(a, s) = Prob(a_t = a, s_t = s)$. We get

$$\lambda_{t+1}(a', s') = \sum_s \sum_{\{a: a'=g(a,s)\}} \lambda_t(a, s) \mathcal{P}(s, s')$$

- ▶ A *stationary* distribution satisfies

$$\lambda(a, s) = \lambda_{t+1}(a, s) = \lambda_t(a, s)$$

INTERPRETATION OF STATIONARY DISTRIBUTION

- ▶ The fraction of time that an infinitely lived agent spends in state (a, s)
- ▶ Fraction of households in state (a, s) in a given period in a stationary equilibrium
- ▶ In the models we study below, the initial distribution of agents over individual state variables (a, s) remains constant over time even though the state of the individual household is a stochastic process

INTRODUCING INCOMPLETE MARKETS AND HETEROGENEITY

- ▶ I will run you through some specific models that build on the above framework
- ▶ In this lecture, the market incompleteness we look at is relatively ad-hoc (in addition to only a non-state contingent bond being available to agents, we introduce a simple exogenous borrowing limit)
- ▶ In the future lectures, we will have more elaborate frictions and but will then typically make the heterogeneity aspect simpler
 - ▶ e.g. one saver and one borrower

PURE CREDIT ECONOMY: HUGGETT (1993)

HUGGET ENVIRONEMNT

- ▶ Two changes to framework above
 1. We determine r in equilibrium
 2. We add a borrowing limit

- ▶ Borrowing limit is exogenously assumed to be

$$a > -\phi$$

- ▶ Could be natural debt limit ($\phi = \frac{ws^1}{r}$) or exogenous parameter that is more restrictive (more on this later)

COMPETITIVE EQUILIBRIUM

- ▶ Given ϕ and \mathcal{P} , a stationary competitive equilibrium is an interest rate r , a policy function $a' = g(a, s)$ and a stationary distribution $\lambda(a, s)$, such that
 1. $g(a, s)$ solves households' maximization problem, taking r as given
 2. $\lambda(a, s)$ is implied by \mathcal{P} and $g(a, s)$
 3. Asset markets clear

$$\sum_{a,s} \lambda(a, s) g(a, s) = 0$$

COMPUTATIONAL SOLUTION ALGORITHM

1. Guess an interest rate r^j
2. Given r^j , solve household's problem for $g^j(a, s)$
3. Use $g^j(a, s)$ and \mathcal{P} to compute $\lambda^j(a, s)$
4. Compute excess demand of savings

$$e^j \equiv \sum_{a,s} \lambda^j(a, s) g^j(a, s)$$

5. If $|e^j| < tol$, stop

Otherwise, if $e^j > 0$, set $r^{j+1} < r^j$; if $e^j < 0$, set $r^{j+1} > r^j$

ADDING CAPITAL: AIYAGARI (1994)

ENVIRONMENT: PRODUCTION ECONOMY

- ▶ Households:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + k_{t+1} = (r_t + 1 - \delta)k_t + w_t s_t$$

(again, I have omitted subscript i for the households' problem)

- ▶ Representative firm:

$$\max AF(K_t, N_t) - r_t K_t - w_t N_t$$

ENVIRONMENT: PRODUCTION ECONOMY

- ▶ Firm FOCs:

$$w_t = A \frac{\partial F}{\partial N_t}$$
$$r_t = A \frac{\partial F}{\partial K_t}$$

- ▶ Household policy function

$$k_{t+1} = g(s_t, k_t)$$

- ▶ A constant, so no aggregate risk

COMPETITIVE EQUILIBRIUM

► Given \mathcal{P} , a stationary competitive equilibrium is a set of prices (r, w) , a policy function $k' = g(k, s)$, a stationary distribution $\lambda(k, s)$ and an aggregate allocation (K, N) such that

1. $g(k, s)$ solves households' maximization problem, taking prices as given
2. $\lambda(k, s)$ is implied by \mathcal{P} and $g(k, s)$
3. Firms maximize profits
4. Labor markets clear

$$N_t = \xi^\infty \bar{s}$$

5. Capital markets clear

$$\sum_{k,s} \lambda(k, s) g(k, s) = K$$

COMPUTATIONAL SOLUTION ALGORITHM

1. Guess a capital stock K^j
2. Given K^j , solve firm's problem for r^j, w^j
3. Given r^j, w^j solve households problem for $g^j(k, s)$
4. Use $g^j(k, s)$ and \mathcal{P} to compute $\lambda^j(k, s)$
5. Calculate implied aggregate capital stock

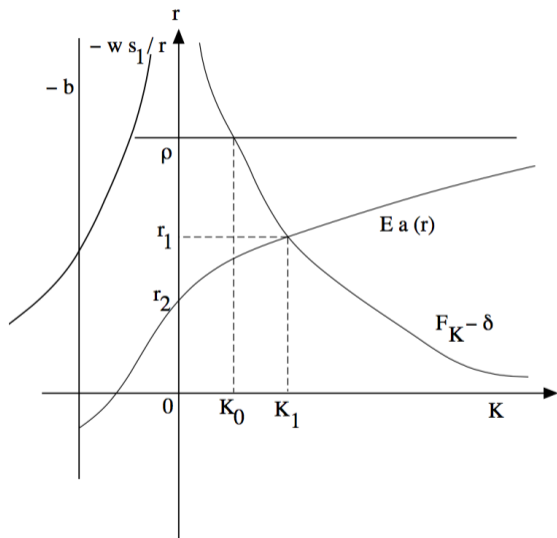
$$\tilde{K}^j \equiv \sum_{k,s} \lambda^j(k, s) g^j(k, s)$$

6. If $|\tilde{K}^j - K^j| < tol$, stop
Otherwise, if $\tilde{K}^j - K^j > 0$, set $K^{j+1} > K^j$; if $\tilde{K}^j - K^j < 0$, set $K^{j+1} < K^j$

PUTTING THINGS TOGETHER

- ▶ In Bewley setting, there is a precautionary savings motive for each individual agent. However, aggregate savings are still zero in equilibrium. Tightening borrowing limit will be reflected in lower interest rates.
- ▶ The Aiyagari setting, there is a positive supply of aggregate savings, because savings are in capital. In this setting, precautionary motives will lead to lower interest rates (returns) and a higher capital stock.
- ▶ Can put things together in one diagram ...

AGGREGATE PRECAUTIONARY SAVINGS



ADDING AGGREGATE RISK: KRUSELL AND SMITH (1998)

AGGREGATE RISK

- ▶ Suppose A_t subject to shocks (aggregate risk)
- ▶ In this case, no invariant distribution $\lambda(k, s)$
- ▶ Since the distribution in a given period matters for prices, each agent will want to know the entire distribution
- ▶ Insight of Krusell and Smith: can approximate distribution with a finite set of moments (the mean)

INSIGHTS OF KRUSELL AND SMITH

- ▶ Not that much lost with a representative agent model
- ▶ Model with uninsurable idiosyncratic risk displays far less dispersion and skewness in wealth than we see in US data
- ▶ The framework remains a very influential baseline in macroeconomics!

THE LATEST GENERATION OF HETEROGENEOUS AGENT
MACRO MODELS

THE WORLD ACCORDING TO HANK

- ▶ As reviewed in Lecture 2, modern quantitative DSGE models usually feature New Keynesian elements (e.g. nominal rigidities) around a neoclassical core
- ▶ The models studied above add substantial complexity to the neoclassical core, through heterogeneity (idiosyncratic risk) and incomplete markets
- ▶ The very latest generation of macro models bridges this more complex core again with New Keynesian features
 - ▶ Heterogeneous Agent New Keynesian (“HANK”) models
 - ▶ See e.g. [Kaplan, Moll, and Violante \(2018\)](#)

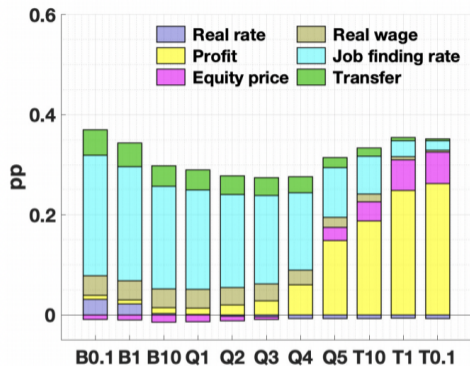
FROM MICRO TO MACRO

- ▶ HANK models allow to study empirically realistic wealth and income distributions
- ▶ A key element is the marginal propensity to consume out of transitory income (the PIH does not hold)
- ▶ Distributional features of the economy:
 - ▶ Are an object of interest
 - ▶ Matter meaningfully for aggregate dynamics

FROM MICRO TO MACRO

- ▶ In the first HANK models, emphasis mostly on matching micro moments
- ▶ More recently, emphasis on also quantitatively matching/explaining aggregate dynamics, e.g. responses to different types of shocks
 - ▶ E.g. Auclert, Rognlie, and Straub (2020) and Bayer, Born, and Luetticke (2020)

A PICTURE FROM A FORMER UMD STUDENT



- ▶ From [Lee \(2020\)](#): “Quantitative Easing and Inequality”
- ▶ The graph shows the welfare effects of QE in terms of consumption equivalents for different income groups and decomposed into different sources

REMARKS AND FURTHER READING

- ▶ In this course, we focus in more detail on financial frictions and will typically have relatively simple layers of heterogeneity
- ▶ Important building blocks of HANK models are related to what we study
 - ▶ Different types of market incompleteness
 - ▶ E.g. often multiple assets with different degrees of liquidity and different returns
- ▶ Highly recommended further reading on the HANK literature:
 - ▶ Ben Moll's teaching material (available on his [website](#))

WRAPPING UP

WRAPPING UP

- ▶ We have introduced **idiosyncratic risk** and **incomplete markets** in combination
- ▶ We got to know some canonical heterogeneous agent models
- ▶ From now on, we will start going deeper into the formulation of financial frictions, and study how they matter for macroeconomic dynamics

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