

EARNINGS-BASED BORROWING CONSTRAINTS  
AND PECUNIARY EXTERNALITIES

Thomas Drechsel    Seho Kim

(University of Maryland)

Advances in Macro Finance  
Tepper-LAEF Conference  
April 8, 2022

## MOTIVATION

- ▶ US firms face two types of credit constraints: asset-based and earnings-based
  - ▶ Liquidation value of physical assets vs. borrower's current EBITDA limit debt access
  - ▶ Direct micro evidence: 80% of corporate debt is earnings-based ([Lian and Ma, 2020](#))
  - ▶ Consequences for business cycle dynamics ([Drechsel, 2020](#))
- ▶ Limited understanding of **normative implications** of earnings-based constraints
  - ▶ This paper provides a theoretical treatment
  - ▶ Structural model with formal welfare characterization
  - ▶ Implications for optimal macroprudential policy

## PREVIEW OF FINDINGS

- ▶ Asset-based constraint: firms **over-borrow** in decentralized equilibrium
  - ▶ Echoes existing insights of the literature, e.g. [Dávila and Korinek \(2018\)](#)
  - ▶ Higher asset price relaxes constraint → not internalized
- ▶ Earnings-based constraint: firms **under-borrow** in decentralized equilibrium
  - ▶ Higher input price (wage) tightens constraint → not internalized
  - ▶ Depending on labor market structure, can also lead to constrained efficiency

## RELATED LITERATURE

- ▶ **Pecuniary externalities with financial frictions:**  
Lorenzoni (2008), Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013), Bianchi (2016), Dávila and Korinek (2018), Ottonello, Perez, and Varraso (2019),...
  - ▶ **Subtleties in the policy implications of different types of credit constraints**
- ▶ **Insights on the specific nature of credit constraints:**  
Lian and Ma (2020), Drechsel (2020), Greenwald (2019),...
  - ▶ **Normative implications of asset-based and earnings-based constraints**

## PLAN FOR THE TALK

1. Main intuition
2. Empirical evidence
3. The model
  - A. Setting
  - B. Efficiency analysis
  - C. Model restrictions and main results
4. Extensions
  - ▶ Working capital, open economy, input vs. output prices
5. Conclusion

## MAIN INTUITION

## MAIN INTUITION

- ▶ Consider a generic financial constraint:

$$\Phi(x', z, \tilde{z}) \geq 0$$

- ▶  $x'$ : financial asset position ( $x' < 0$ : borrowing)
  - ▶  $z$ : endogenous variables chosen by the agent
  - ▶  $\tilde{z}$ : endogenous or exogenous variables taken as given by the agent (e.g., prices)
- ▶ Agents' choices move prices in  $\tilde{z} \rightarrow$  pecuniary externality
- ▶ The direction of price changes matters for normative implications

## MAIN INTUITION: ASSET-BASED CONSTRAINT

- ▶ Asset-based collateral constraint:
  - ▶  $z = k'$  ,  $\tilde{z} = q$ , and  $\Phi(x', z, \tilde{z}) = x' + \phi q k' \geq 0 \Rightarrow -x' \leq \phi q k'$
  - ▶  $q = q(X, K)$ : market price of capital as a function of the aggregate state variables
  - ▶ Aggregate states are net worth and capital
- ▶ Suppose  $q$  depends positively on net worth
  - ▶ If more borrowing today:
    - $\Rightarrow$  Future aggregate borrower net worth  $\downarrow$
    - $\Rightarrow$  Future price of capital  $\downarrow$  – *through lower demand for capital*
    - $\Rightarrow$  Tightening of future borrowing limit
- ▶ Agents do not internalize this effect, **over-borrow** relative to the social optimum



## MAIN INTUITION: EARNINGS-BASED CONSTRAINT

- ▶ Earnings-based constraint:

- ▶  $z = [y, \ell]$  ,  $\tilde{z} = w$ , and  $\Phi(x', z, \tilde{z}) = x' + \tilde{\phi}(y - w\ell) \geq 0 \Rightarrow -x' \leq \tilde{\phi}(y - w\ell)$

- ▶  $w = w(X, K)$ : market wage as a function of the aggregate state variables

- ▶ Suppose  $w$  increases with net worth

- ▶ If more borrowing today:

- ⇒ Future aggregate borrower net worth ↓

- ⇒ Future wage ↓ – *through lower supply of labor*

- ⇒ Loosening of future borrowing limit

- ▶ Agents do not internalize this effect, **under-borrow** relative to the social optimum

## EMPIRICAL EVIDENCE

## EMPIRICAL EVIDENCE

- ▶ Mounting microeconomic evidence in favor of  $-x' \leq \tilde{\phi}(y - wl)$
- ▶ Earnings-based borrowing constraints can arise through:
  - ▶ Debt covenants: legal provisions in loan contracts
  - ▶ Credit ratings, bankruptcy procedures, ...
- ▶ Lian and Ma (2020): 80% of corporate debt earnings-based
- ▶ Drechsel (2020): earnings-based constraints matter for business cycle dynamics
- ▶ Caglio, Darst, and Kalemli-Özcan (2021) shows that earnings-based are prevalent for private small and medium-sized companies (SMEs)

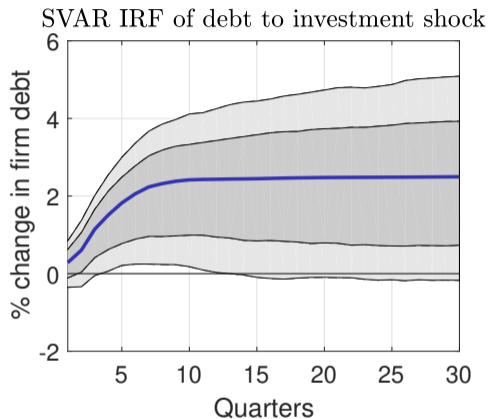
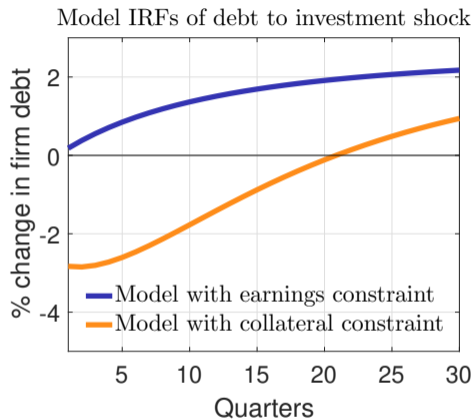
## EMPIRICAL EVIDENCE (DRECHSEL, 2020)

	<b>Covenant type</b>	<b>Median</b>	<b>Mean</b>	<b>Freq. (%)</b>
1	Max Debt to EBITDA	3.75	4.60	60.5
2	Min EBITDA to Interest	2.50	2.56	46.7
3	Min EBITDA to Fixed Charge	1.25	1.42	22.1
4	Max. Leverage ratio	0.60	0.64	21.3
5	Max. Capex	20M	194M	15.1
6	Net Worth	126M	3.2B	11.5

EBITDA is *earnings before interest, taxes, depreciation and amortization*

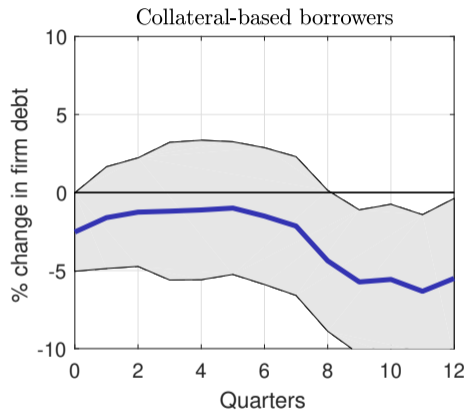
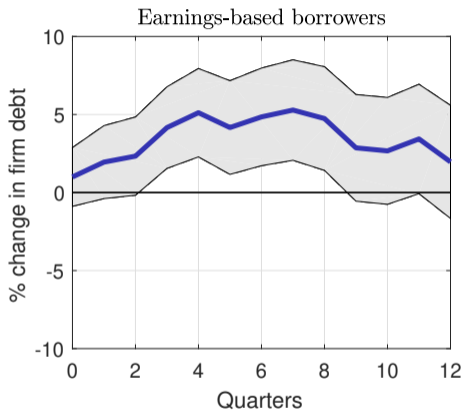
- ▶ Covenants based on earnings very prevalent
- ▶ Covenants bind frequently with large economic effects (see e.g. Chodorow-Reich and Falato, 2021)

## BUSINESS CYCLE CONSEQUENCES (DRECHSEL, 2020)



- ▶ Aggregate debt response consistent with earnings-based constraint, not with collateral constraint

## BUSINESS CYCLE CONSEQUENCES (DRECHSEL, 2020)



- ▶ Split of debt response across borrower types consistent with model prediction across alternative constraints

## THE MODEL

## SETTING

- ▶ Build on structure [Dávila and Korinek \(2018\)](#) + [labor market](#)
- ▶ Three period model ( $t = 0, 1, 2$ )
- ▶ The state of nature,  $\theta \in \Omega$ , is revealed at date 1
- ▶ Two types of agents: borrowers ( $b$ ) and lenders ( $l$ )
- ▶ Both agents produce, consume and supply labor
- ▶ Borrowers face credit constraints



## AGENTS' PROBLEM

- Agent  $i \in \{b, l\}$  maximizes

$$U^i = \mathbb{E}_0 \left[ \sum_{t=0}^2 \beta^t u^i(c_t^i, \ell_{st}^i) \right]$$

subject to budget constraints

$$c_0^i + h^i(k_1^i) + \int_{\theta \in \Theta} m_1^\theta x_1^{i,\theta} d\theta = e_0^i$$

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta}, \quad \forall \theta$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta}, \quad \forall \theta$$

and financial constraints

$$\Phi_1^b(x_1^b, k_1^b) \geq 0$$

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_1^b, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0, \quad \forall \theta$$

## FINANCIAL CONSTRAINT

- ▶ Main constraint of interest: period-1 financial constraint

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_1^b, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0, \forall \theta$$

- ▶ General formulation in which all model variables can enter
- ▶ Includes:

- ▶ Asset-based constraint:  $-x_2^{b,\theta} \leq \phi q^\theta k_2^{b,\theta}$
- ▶ Earnings-based constraint:  $-x_2^{b,\theta} \leq \tilde{\phi}(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta})$
- ▶ Interest coverage constraint:  $-x_2^{b,\theta} \leq \hat{\phi} \frac{F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta}}{i_2^\theta}$

## SOLVING THE MODEL

- ▶ Decentralized equilibrium (backward induction)
  - ▶ Date 2: purely intra-temporal consumption, labor supply and demand
  - ▶ Date 1: express welfare as a function of state variables

$$V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1) = \max_{\{c_1^{i,\theta}, c_2^{i,\theta}, k_2^{i,\theta}, x_2^{i,\theta}, \ell_{dt}^{i,\theta}, \ell_{st}^{i,\theta}\}} \left\{ u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta}) \right\}$$

s.t. period 1 and 2 budget constraint and period 1 financial constraint

- ▶ net worth:  $n_1^{i,\theta} \equiv e_1^{i,\theta} + x_1^{i,\theta}$
- ▶ Prices are functions of only aggregate states
- ▶ In equilibrium,  $n_1^{i,\theta} = N_1^{i,\theta}$ ,  $k_1^i = K_1^i$

## SUFFICIENT STATISTICS

- ▶ Following [Dávila and Korinek \(2018\)](#), “sufficient statistics” approach
- ▶ The effect of changes in  $N_1^{j,\theta}$  on  $V^{i,\theta}$ :

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta} \mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta} \mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta} \mathcal{C}_{N^j}^{i,\theta}$$

- ▶ Welfare changes that are not internalized by the agents, work through prices
- ▶ Distinguish between distributive effects ( $\mathcal{D}$ ) and constraint effects ( $\mathcal{C}$ )

## SUFFICIENT STATISTICS

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta} \mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta} \mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta} \mathcal{C}_{N^j}^{i,\theta}$$

► **Distributive effects:**

- Changes in prices that benefit one agent, make other agent worse off
- Not our focus

## SUFFICIENT STATISTICS

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta} \mathcal{D}_{1N_j}^{i,\theta} + \lambda_2^{i,\theta} \mathcal{D}_{2N_j}^{i,\theta} + \kappa_2^{i,\theta} \mathcal{C}_{N_j}^{i,\theta}$$

- **Constraint effects:** changes in prices that affect tightness of credit constraints

$$\mathcal{C}_{N_j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta} \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}$$
$$\mathcal{C}_{N_j}^{l,\theta} = 0$$

( $\kappa_2^{i,\theta}$  is Lagrange multiplier on the financial constraint)

## OTHER EFFECTS

- ▶ In the same vein, can study effects coming from  $\frac{dV^{i,\theta}(\cdot)}{dK_1^{j,\theta}}$
- ▶ We focus on over-/under-borrowing rather than over-/under-investing
- ▶ Bound by “anything goes” result of [Dávila and Korinek \(2018\)](#)

## EFFICIENCY ANALYSIS



## CONSTRAINED EFFICIENT ALLOCATION

- ▶ Planner internalizes distributive and constraint effects of borrowing decision
- ▶ Chooses allocations in  $t = 0$  subject to:
  1. The same  $t = 0$  constraints as the private agents
  2. The optimal behavior of private agents in periods  $t = 1, 2$
- ▶ Corresponds to problem of constrained Ramsey planner who can levy  $t = 0$  taxes

Details

## IMPLEMENTATION OF CONSTRAINED EFFICIENT ALLOCATION

- ▶ **Proposition:** A decentralized equilibrium with the following corrective taxes replicates the constrained efficient allocation

$$\tau_x^{i,\theta} = -\Delta MRS_{01}^{ij,\theta} \mathcal{D}_{1N^i}^{i,\theta} - \Delta MRS_{02}^{ij,\theta} \mathcal{D}_{2N^i}^{i,\theta} - \tilde{\kappa}_2^{b,\theta} \mathcal{C}_{N^i}^{b,\theta}, \quad \forall i, \theta$$

- ▶  $\tau_x^{i,\theta} > 0$ : taxes on saving  $\Rightarrow$  under-borrowing in decentralized equilibrium
- ▶  $\Delta MRS_{0t}^{ij,\theta} \equiv MRS_{0t}^{i,\theta} - MRS_{0t}^{j,\theta}$
- ▶  $\tilde{\kappa}_2^{b,\theta}$ : shadow price on credit constraint

## HOW TO PROCEED WITH EFFICIENCY ANALYSIS

- ▶ For specific financial constraints  $\Phi_2^{b,\theta}$ , find  $C_{Ni}^{b,\theta}$
- ▶ Given sign of  $C_{Ni}^{b,\theta}$ , determine sign of  $\tau_x^{i,\theta}$ 
  - ▶ If  $\tau_x^{i,\theta} < 0$ : planner corrects 'over-borrowing'
  - ▶ If  $\tau_x^{i,\theta} > 0$ : planner corrects 'under-borrowing'
- ▶ To pin down signs, need to specialize model further

## ADDITIONAL MODEL RESTRICTIONS AND MAIN RESULTS

## ADDITIONAL MODEL RESTRICTIONS

▶ **Condition for collateral constraints:**

$$\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i$$

▶ **Explanation:**

- ▶ Supply of capital is predetermined
- ▶ An increase in net worth raises the demand for capital  
⇒ upward pressure on capital price

## ADDITIONAL MODEL RESTRICTIONS

- ▶ **Condition for earnings-based constraints:**

$$\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0, \quad \forall i$$

- ▶ **Argument:**

- ▶ Demand for labor is pinned down conditional on capital
- ▶ Higher net worth increases consumption  $\Rightarrow$  (under standard preference) demand for leisure  $\uparrow$ , so decrease in labor supply  
 $\Rightarrow$  upward pressure on wage

## ADDITIONAL MODEL RESTRICTIONS

▶ **Condition for interest coverage constraints:**

$$\frac{\partial m_2^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i$$

$$\frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i$$

▶ **Argument:**

- ▶ Higher net worth increases incentive to save more to smooth consumption  
⇒ Price of debt (= inverse of interest rate) increase (tends to move in the same way with the price of capital due to no-arbitrage restriction)
- ▶ Direct analogy to the argument for the period 1 wage

## MAIN RESULTS

▶ **Collateral constraint:**

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi q^\theta k_2^{b,\theta} \geq 0$$

▶ **Proposition:** There is an **over-borrowing** effect through constraint externalities

▶ **Proof:**

$$\text{▶ } -\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} = -\tilde{\kappa}_2^{b,\theta} \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{i,\theta}} \leq 0 \Rightarrow \text{subsidize saving (= penalize borrowing)}$$



## MAIN RESULTS

► **Earnings-based constraint:**

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \tilde{\phi}(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta}) \geq 0$$

► **Proposition:** There is an **under-borrowing** effect through constraint externalities

► **Proof:**

►  $-\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} = -\tilde{\kappa}_2^{b,\theta} \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0 \Rightarrow$  penalize saving (= subsidize borrowing)

► **Note:** if labor supply inelastic,  $\partial w / \partial N$  term drops out  $\Rightarrow$  **constrained efficiency**

## MAIN RESULTS

► **Interest coverage constraint:**

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \hat{\phi} \frac{F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta}}{i_2^\theta} \geq 0$$

► **Proposition:** There is an **ambiguous** effect through constraint externalities

► **Proof:**

►  $-\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} = -\tilde{\kappa}_2^{b,\theta} \left( \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial i_2^\theta} \frac{\partial i_2^\theta}{\partial N_1^{i,\theta}} \right) \leq 0$

► This constraint is “mixture” of earnings-based and asset-based constraint

► Why?  $1/i$  co-moves with  $q$  through no-arbitrage condition

## SUMMARY OF FINDINGS

- ▶ Asset-based constraint: agents **over-borrow** in decentralized equilibrium
  - ▶ Echoes existing insights of the literature, e.g. [Dávila and Korinek \(2018\)](#)
  - ▶ Higher asset price relaxes constraint → not internalized
- ▶ Earnings-based constraint: agents **under-borrow** in decentralized equilibrium
  - ▶ Higher input price (wage) tightens constraint → not internalized
- ▶ Interest coverage constraint: 'mixture' of earnings- and asset-based constraint

## EXTENSIONS

## WORKING CAPITAL

- ▶ Several authors propose models with working capital and collateral constraints
  - ▶ See e.g. [Bianchi and Mendoza \(2010\)](#), [Jermann and Quadrini \(2012\)](#), [Bianchi \(2016\)](#)
- ▶ Suppose wage bill financed with an intraperiod loan  $x_{wc} = -\psi w\ell$

$$-(x' - \psi w\ell) \leq \tilde{\phi}(F(k, \ell) - w\ell) \Rightarrow -x' \leq -(\tilde{\phi}F(k, \ell) - (\tilde{\phi} + \psi)w\ell)$$

- ▶  $\tilde{\phi} + \psi > \tilde{\phi}$ : more pronounced under-borrowing effect

## SMALL OPEN ECONOMY

- ▶ Several papers on welfare consequences of borrowing constraints in small open economies (see e.g. [Bianchi, 2011](#))
- ▶ We focus on an endogenous interest rate because the background on earnings-based constraints is largely provided for the U.S.
- ▶ Microeconomic evidence on the specific forms of constraints is thinner for emerging economies, but would be very welcome
- ▶ Note that a fixed interest rates would make interest coverage constraint inherit the consequences of the earnings-based constraint (no 'mixture' result)

## OUTPUT VS. INPUT PRICES

- ▶ In our real model,  $w$  denotes relative price
- ▶ But what if final goods price is not equal to 1?
- ▶ Need multi-good environment to think about meaningful output price variation
  1. Monopolistically competitive firms environment
    - ▶ Prices are choice variables, so firms internalize how price affects the constraint
    - ▶ However, firms would not internalize how their individual choices affect aggregate inflation, which could affect nominal debt limits
  2. Perfectly competitive firms environment
    - ▶ Effects on relative prices between different goods not internalized? (Fazio (2021))

## CONCLUSION



## CONCLUSION

- ▶ Whether debt is backed by collateral or linked to firms' earnings has sharply different implications for macroprudential policy
- ▶ The pecuniary externality through wages in earnings-based constraints prescribes that a regulatory authority should, if anything, encourage firms to borrow
- ▶ Our analysis highlights the importance of a proper understanding of the microeconomic details behind which constraints matter in which markets
  - ▶ Asset-based borrowing: mortgage markets, repo markets, ...
  - ▶ Earnings-based borrowing: corporate credit markets

## REFERENCES

- BENIGNO, G., H. CHEN, C. OTROK, A. REBUCCI, AND E. R. YOUNG (2013): "Financial crises and macro-prudential policies," *Journal of International Economics*, 89, 453–470.
- BIANCHI, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101, 3400–3426.
- (2016): "Efficient Bailouts?" *American Economic Review*, 106, 3607–59.
- BIANCHI, J. AND E. G. MENDOZA (2010): "Overborrowing, Financial Crises and 'Macro-prudential' Taxes," Working Paper 16091, NBER.
- CAGLIO, C. R., R. M. DARST, AND S. KALEMLI-ÖZCAN (2021): "Risk-Taking and Monetary Policy Transmission: Evidence from Loans to SMEs and Large Firms," Tech. rep., National Bureau of Economic Research.
- CHODOROW-REICH, G. AND A. FALATO (2021): "The Loan Covenant Channel: How Bank Health Transmits to the Real Economy," *The Journal of Finance*, n/a.
- DÁVILA, E. AND A. KORINEK (2018): "Pecuniary Externalities in Economies with Financial Frictions," *The Review of Economic Studies*, 85, 352–395.
- DRECHSEL, T. (2020): "Earnings-based borrowing constraints and macroeconomic fluctuations," *Working paper*.
- FAZIO, M. (2021): "Financial Stabilisation Policies in a Credit Crunch: Zombie Firms and the Effective Lower Bound," *Working paper, London School of Economics*.
- GREENWALD, D. (2019): "Firm debt covenants and the macroeconomy: The interest coverage channel," *Working paper*.
- JERMANN, U. AND V. QUADRINI (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102, 238–71.
- LIAN, C. AND Y. MA (2020): "Anatomy of Corporate Borrowing Constraints\*," *The Quarterly Journal of Economics*, 136, 229–291.
- LORENZONI, G. (2008): "Inefficient Credit Booms," *The Review of Economic Studies*, 75, 809–833.
- OTTONELLO, P., D. J. PEREZ, AND P. VARRASO (2019): "Are Collateral-Constraint Models Ready for Macroprudential Policy Design?" *Unpublished Manuscript*.

## APPENDIX SLIDES

## FORMAL SOCIAL PLANNER PROBLEM

$$\max \sum_i \alpha^i \{u^i(C_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)]\}$$

subject to  $t = 0$  resource and credit constraints

$$\sum_i [C_0^i + h^i(K_1^i) - e_0^i] \leq 0$$

$$\sum_i X_1^{i,\theta} = 0, \quad \forall \theta$$

$$\Phi_1^i(X_1^i, K_1^i) \geq 0, \quad \forall i$$